Regular non-Abelian vacua in $\mathcal{N} = 4$, SO(4) gauged supergravity

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We present a family of globally regular $\mathcal{N} = 1$ vacua in the D=4, $\mathcal{N} = 4$ gauged supergravity of Gates and Zwiebach. These solutions are labeled by the ratio ξ of the two gauge couplings, and for $\xi = 0$ they reduce to the supergravity monopole previously used for constructing the gravity dual of $\mathcal{N} = 1$ super Yang-Mills theory. For $\xi > 0$ the solutions are asymptotically anti de Sitter, but with an excess of the solid angle, and they reduce exactly to anti de Sitter for $\xi = 1$. Solutions with $\xi < 0$ are topologically $R^1 \times S^3$, and for $\xi = -2$ they become $R^1 \times S^3$ geometrically. All solutions with $\xi \neq 0$ can be promoted to D=11 to become vacua of M-theory.

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Regular supersymmetric backgrounds in gauged supergravities (SUGRA) play an important role in the context of the AdS/CFT correspondence (see [1] for a review). Upon uplifting to higher dimensions they become vacua of string/M theory and can be used for the dual description of strongly coupled gauge field theories. In this way, for example, the monopole solution [2] of the $\mathcal{N} = 4$ gauged SUGRA has given rise to the holographic interpretation of confining $\mathcal{N} = 1$ super Yang-Mills (SYM) theory [3]. Constructing such solutions, however, is rather involved. This is why, despite their importance, very few regular vacua of gauged SUGRA's are known.

In this Letter we present a family of globally regular $\mathcal{N} = 1$ vacua that contains the monopole solution of Ref. [2] as special case. We work in the context of the $\mathcal{N} = 4$ gauged SUGRA in four dimensions. This theory exists in two inequivalent versions: the SU(2)×SU(2) model of Freedman and Schwarz (FS) [4], whose solutions were studied in [2], and the SO(4) model of Gates and Zwiebach (GZ) [5]. Both models contain in the bosonic sector the graviton $g_{\mu\nu}$, dilaton ϕ, axion $\mathbf{a}$, and two non-Abelian gauge fields A^a_μ and B^a_μ with gauge couplings e_A and e_B and with gauge group SU(2)×SU(2). The important difference

between the two models is that in the FS model the dilaton potential has no stationary points, while in the GZ model one has (when $\mathbf{a} = 0$)

$$\mathrm{U}(\phi) = -\frac{e_A^2}{8}\,(e^{-2\phi} + \xi^2 e^{2\phi} + 4\xi). \tag{1}$$

This potential does have stationary points, and, depending on the sign of $\xi \equiv e_B/e_A$, its extremal value – the cosmological constant – can be positive or negative. If one sets $\mathbf{a} = B^a_\mu = \xi = 0$, then the FS and GZ models coincide and admit as a solution the $\mathcal{N} = 1$ vacuum of Ref. [2] – the Chamseddine-Volkov (CV) monopole. If $\mathbf{a} = B^a_\mu = 0$ but $\xi \neq 0$, then the two models are no longer the same, and we find that within the GZ model the CV monopole admits generalizations for any $\xi \neq 0$. These solutions are topologically different from the CV monopole, although approach the latter pointwise as $\xi \to 0$. They can be uplifted to D=11, which may suggest a holographic interpretation for them.

We consider the $\mathbf{a} = B^a_\mu = 0$ truncation of the GZ model whose bosonic sector is described by the Lagrangian

$$\mathcal{L} = \frac{1}{4}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\,e^{2\phi}F^a_{\mu\nu}F^{a\mu\nu} - \mathrm{U}(\phi). \tag{2}$$

Here $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon_{abc}A^b_\mu A^c_\nu$ with $a = 1, 2, 3$, the scale is chosen such that $e_A = 1$, and $\mathrm{U}(\phi)$ is given by (1). Consistency of setting the axion to zero requires that $*F^a_{\mu\nu}F^{a\mu\nu} = 0$. The theory contains also fermions: the gaugino χ and gravitino ψ_μ, whose SUSY variations for a purely bosonic background are

$$\begin{aligned}\delta\chi &= \frac{1}{\sqrt{2}}\,\gamma^\mu\partial_\mu\phi\,\epsilon + \frac{1}{2}\,e^\phi\mathcal{F}\epsilon + \frac{1}{4}\,(e^{-\phi} - \xi e^\phi)\epsilon, \\ \delta\psi_\mu &= \mathcal{D}_\mu\epsilon + \frac{1}{2\sqrt{2}}\,e^\phi\mathcal{F}\gamma_\mu\epsilon + \frac{1}{4\sqrt{2}}\,(e^{-\phi} + \xi e^\phi)\gamma_\mu\epsilon.\end{aligned} \tag{3}$$

Here $\mathcal{F} = \frac{1}{2}\alpha^a F^a_{\alpha\beta}\gamma^\alpha\gamma^\beta$ and $\mathcal{D}_\mu = \partial_\mu + \frac{1}{4}\,\omega_{\alpha\beta,\mu}\gamma^\alpha\gamma^\beta - \frac{1}{2}A^a_\mu\alpha^a$; the late (μ,ν) and early (α,β) Greek letters correspond to the spacetime and tangent space indices, respectively. The gamma matrices are subject to $\frac{1}{2}(\gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha) = \eta_{\alpha\beta} \equiv \mathrm{diag}(-1, 1, 1, 1)$. Introducing Pauli matrices of four different types, σ^a, $\underline{\sigma}^b$, τ^c, $\underline{\tau}^d$, which act in four different spaces, respectively (such that, for example, $[\sigma^a, \underline{\sigma}^b] = 0$), one can choose $\gamma^\alpha \equiv (\gamma^0, \gamma^a) = (i\underline{\sigma}^1, \underline{\sigma}^2\sigma^a)$. The gauge group SU(2)×SU(2) is generated by the antihermitean matrices α^a and β^b, $[\alpha^a, \beta^b] = 0$, $\alpha^a\alpha^b = -\epsilon_{abc}\alpha^c - \delta_{ab}$, and similarly for β^a. One can choose $\alpha^a = i\tau^a$, $\beta^a = i\underline{\tau}^a$. The generators β^a correspond to the field B^a_μ that is truncated to zero.

We wish to study fields that preserve some of the supersymmetries, in which case $\delta\chi = \delta\psi_\mu = 0$ for certain $\epsilon \neq 0$. We restrict to the static and spherically symmetric sector parameterized by coordinates $(t, \rho, \vartheta, \varphi)$ with

$$ds^2_{(4)} = -e^{2V(\rho)}dt^2 + e^{2\lambda(\rho)}d\rho^2 + r^2(\rho)d\Omega^2, \tag{4}$$

$$\tau^a A^a_\mu dx^\mu = \frac{i}{2}(1 - w(\rho))[T, dT], \qquad \phi = \phi(\rho). \tag{5}$$

Here, with $n^a = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$, one has $d\Omega^2 = dn^a dn^a$ and $T = \tau^a n^a$. Imposing the isotropic gauge condition, $r = \rho e^\lambda$, the spatial part of the metric becomes conformally flat, $ds^2_{(3)} = e^{2\lambda}dx^a dx^a$, with $x^a = \rho n^a$. Choosing the tetrad $\theta^\alpha = (e^V dt, e^\lambda dx^a)$, the spin connection is obtained from $d\theta_\alpha + \omega_{\alpha\beta} \wedge \theta^\beta = 0$. Setting in (3) $\delta\chi = \delta\psi_\mu = 0$ gives then the equations for the SUSY Killing spinors ϵ:

$$\begin{aligned} 0 &= 2\sqrt{2}\,\mathrm{e}^{-\lambda}\phi'\,\underline{\sigma}^2\,(\vec{n}\vec{\sigma})\epsilon + 2\,\mathrm{e}^\phi\,\mathcal{F}\epsilon + (\mathrm{e}^{-\phi} - \xi e^\phi)\epsilon\,, \\ 0 &= 2i\sqrt{2}e^{-V}\underline{\sigma}^1\partial_t\epsilon + \sqrt{2}\,e^{-\lambda}(V-\phi)'\underline{\sigma}^2(\vec{n}\vec{\sigma})\epsilon + \xi e^\phi\epsilon, \\ 0 &= \vec{\nabla}\epsilon + \frac{i}{2}\,\lambda'\,(\vec{n}\times\vec{\sigma})\epsilon + \frac{i}{2}\frac{w-1}{\rho}\,(\vec{n}\times\vec{\tau})\epsilon \\ &+ \frac{e^\lambda}{4\sqrt{2}}(2\mathrm{e}^\phi\mathcal{F} + \mathrm{e}^{-\phi} + \xi e^\phi)\underline{\sigma}^2\vec{\sigma}\epsilon. \end{aligned} \tag{6}$$

Here $\mathcal{F} = -r^{-2}\left[\rho w'(\vec{\sigma}\vec{\tau}) + (w^2 - 1 - \rho w')(\vec{n}\vec{\sigma})(\vec{n}\vec{\tau})\right]$ and $' \equiv \frac{d}{d\rho}$; also the usual operations for Euclidean 3-vectors are assumed, for example $\vec{n}\vec{\sigma} \equiv n^a\sigma^a$ and $\vec{\nabla} \equiv \partial/\partial x^a$. Eqs. (6) comprise an overdetermined system of 80 equations for 16 components of ϵ, whose consistency conditions we shall now study.

Let ψ_A, χ_A, $\underline{\psi}_A$, $\underline{\chi}_A$ be eigenspinors of σ^3, τ^3, $\underline{\sigma}^1$, $\underline{\tau}^2$, respectively, with the eigenvalues $(-1)^A$, $A = 1, 2$. We make the ansatz $\epsilon = \epsilon_{AB}$ with $A, B = 1, 2$ and

$$\epsilon_{AB} = \mathcal{U}\exp(i\Psi t)[\Phi_-(\rho) + \Phi_+(\rho)\,\underline{\sigma}^2(\vec{n}\vec{\sigma})]\,\epsilon_0\,\underline{\psi}_A\underline{\chi}_B. \tag{7}$$

Here $\Psi = \frac{1}{2\sqrt{2}}(-1)^{A+B}\xi Q\,\underline{\tau}^2$ where Q is a real constant, $\mathcal{U} = \exp(-\frac{i}{2}\tau_3\varphi)\exp(-\frac{i}{2}\tau_2\vartheta)$, and $\epsilon_0 = \psi_1\,\chi_2 - \psi_2\chi_1$. In fact, ϵ_{AB} is the most general spinor whose total angular momentum, including the orbital part plus spin plus isospin, is zero. Inserting (7) into (6), the variables decouple, and the system reduces to six linear algebraic and two ordinary differential equations for $\Phi_\pm$:

$$A^\pm_{(m)}\Phi_\pm = B^\mp_{(m)}\Phi_\mp, \qquad \Phi'_\pm - \mathcal{A}\Phi_\pm = \mathcal{B}\Phi_\mp\ . \tag{8}$$

Here the coefficients $A^{\pm}_{(m)}$, $B^{\pm}_{(m)}$ $(m = 1, 2, 3)$, $\mathcal{A}$ and $\mathcal{B}$ are functions of the background amplitudes V, λ, r, w, ϕ and their derivatives. The algebraic equations can have a nontrivial solution if only their coefficients fulfill the conditions $A^{+}_{(m)}A^{-}_{(n)} = B^{+}_{(m)}B^{-}_{(n)}$, of which only 5 are independent. Introducing $N = \rho\lambda' + 1$, these 5 conditions are equivalent to the first 5 of the following 6 relations:

$$V' - \phi' = \xi\,\frac{P}{\sqrt{2}N}e^{\phi+\lambda}, \qquad Q = e^{V+\phi}\frac{w}{N}, \tag{9}$$

$$\phi' = \sqrt{2}\,\frac{BP}{N}\,e^{\lambda}, \qquad w' = -\frac{rwB}{N}e^{-\phi+\lambda}, \tag{10}$$

$$N = \sqrt{w^2 + P^2}, \qquad r' = Ne^{\lambda}. \tag{11}$$

Here $P = e^{\phi}\frac{1-w^2}{\sqrt{2}r} + \frac{r}{2\sqrt{2}}(e^{-\phi} + \xi e^{\phi})$ and $B = -\frac{P}{\sqrt{2}r} + \frac{1}{2}e^{-\phi}$. These relations impose nonlinear differential constraints on the background functions ϕ, w, V, λ and the parameter Q. Remarkably, although we have in (9) two equations for the same function $V(\rho)$, the first of these equations is in fact a differential consequence of the second one, and so the system is not overdetermined. The last equation in (11), added for the later convenience, is the identity (in the isotropic gauge used) implied by the definition of N. If Eqs.(9)–(11) are fulfilled, the algebraic equations in (8) are consistent with each other and express Φ_- in terms of Φ_+. Inserting this to the first differential constraint in (8) gives a linear differential equation for $\Phi_+(\rho)$, whose solution can be expressed in quadratures. The second differential constraint in (8) then turns out to be fulfilled identically, by virtue of Eqs. (9)–(11).

The Bogomolnyi equations (9)–(11) therefore provide the full set of consistency conditions that guarantee the existence of SUSY Killing spinors. One can now pass in these equations to an arbitrary gauge by treating $\lambda(\rho)$ as a free function subject to a gauge condition, while considering the second relation in (11) as the dynamical equation for the Schwarzschild radial function $r(\rho)$. Finding then $\Phi_\pm$ gives the spinor ϵ_{AB} for each choice of A, B, which finally corresponds to four independent SUSY Killing spinors, that is to $\mathcal{N} = 1$.

Introducing $y^1 = w$, $y^2 = \phi$, $y^3 = V$, $y^4 = V + \ln r$, the Bogomolnyi equations can also be written as

$$Y^n \equiv \frac{dy^n}{d\rho} - G^{nm}\frac{\partial \mathcal{W}}{\partial y^m} = 0, \qquad m, n = 1, 2, 3, 4, \tag{12}$$

where the target space metric is defined by $G_{mn} = r^2\,e^{V-\lambda}\mathrm{diag}(2e^{2\phi}/r^2, 1, 1, -1)$ and the superpotential is $\mathcal{W} = -re^{V}N$ with N given by (11). We note also that inserting the ansatz (4),(5) to the Lagrangian (2) and integrating over the angles gives $\int \mathcal{L}\sqrt{-g}\,d\vartheta d\varphi =$

$4\pi G_{mn}Y^m Y^n$+total derivative. It then follows that solutions of Eqs. (12) are stationary points of the action.

To integrate the Bogomolnyi equations (9)–(11), the problem actually reduces to studying the closed subsystem (10)–(11) for ϕ, w, r, since V can be obtained afterwards from (9). It seems that these equations can be resolved analytically only for some special values of ξ, and we shall therefore resort to numerical analysis to study the generic case. First of all, we notice the following symmetry of the equations: if ξ, $\phi(\rho)$, $w(\rho)$, $r(\rho)$, $\lambda(\rho)$ is a solution for some value of ξ, then, for any ϵ,

$$e^{-2\epsilon}\xi, \quad \phi(\rho)+\epsilon, \quad w(\rho), \quad e^{\epsilon}r(\rho), \quad \lambda(\rho)+\epsilon \tag{13}$$

is also a solution. To fix this symmetry, we impose the condition $\phi_0 = 0$, with ϕ_0 being the value of ϕ at $r = 0$. Since the sign of ξ is invariant under (13), there are three separate cases to study: $\xi > 0$, $\xi < 0$, and $\xi = 0$.

$\underline{\xi = 0.}$ Eqs. (10)–(11) reduce in this case to the system previously studied in the context of the half-gauged FS model [2]. Its solution is the CV monopole:

$$w = \frac{\rho}{\sinh\rho}, \qquad \frac{re^{-\phi}}{\sqrt{2}} = \frac{\rho e^{-2\phi}}{w} = \sqrt{2\rho\coth\rho - w^2 - 1}, \tag{14}$$

and $e^{\lambda} = \sqrt{2}e^{\phi}$. In this case one has $\phi \sim \rho$ as $\rho \to \infty$.

$\underline{\xi > 0.}$ Choosing the Schwarzschild gauge, $\rho = r$, the essential equations are given by (10),(11) with $e^{\lambda} = 1/N$. They determine $\phi(r)$, $w(r)$, while (9) gives $e^{2V} = Q^2N^2e^{-2\phi}/w^2$. We are interested in everywhere regular solutions, in which case $\phi = O(r^2)$, $w = 1 + O(r^2)$, $N = 1 + O(r^2)$ for $r \to 0$. For $r \to \infty$ one has

$$\begin{aligned} \phi &= -\frac{\ln\xi}{2} + \frac{b}{r} + O(r^{-2}), \quad w = w_* - \frac{w_* b}{r} + O(r^{-2}), \\ N^2 &= 1 + \frac{1}{2}\xi b^2 - \frac{2b(w_*^2-1)}{r} + \frac{\xi r^2}{2} + O(r^{-2}), \end{aligned} \tag{15}$$

where b and w_* are integration constants. The numerical integration of the equations reveals for every value of $\xi > 0$ a global solution $\phi(r)$, $w(r)$ with such boundary conditions in the interval $r \in [0, \infty)$; see Fig.I. For all these solutions the dilaton varies in the finite range and runs into the stationary point of U(ϕ) for $\rho \to \infty$. As $\xi \to 0$, the asymptotic value of ϕ tends to infinity and the solutions approach pointwise the CV monopole (14).

For $\xi = 1$ the solution can be obtained analytically: $\phi(r) = 0$, $w(r) = 1$. Choosing $Q = 1$ (the value of Q can be adjusted by rescaling the time), the metric assumes the standard anti

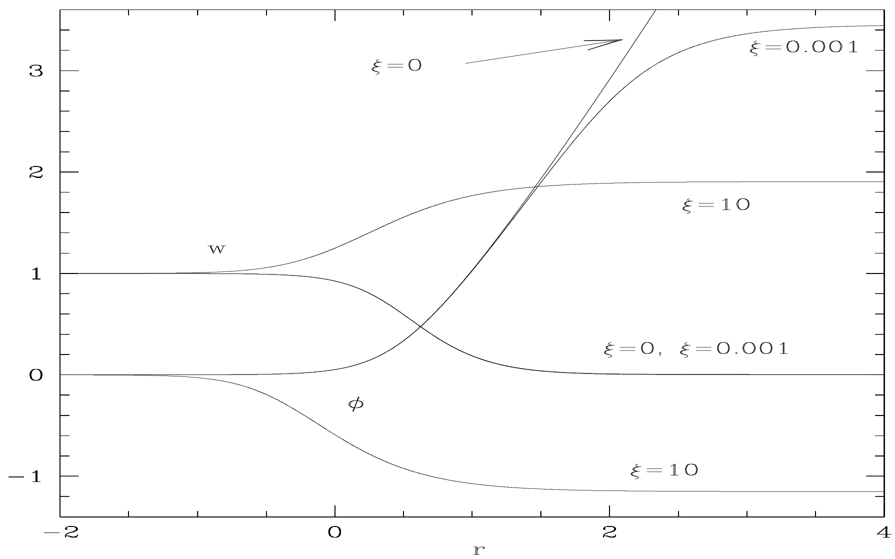

FIG. 1: Globally regular solutions with $\xi > 0$. For $\xi = 0.001$ and $\xi = 0$ the amplitudes w are almost identical.

de Sitter (AdS) form, $ds^2 = -N^2dt^2 + dr^2/N^2 + r^2d\Omega^2$ with $N^2 = 1 + \frac{r^2}{2}$, while the gauge field vanishes. This solution actually has $\mathcal{N} = 4$ supersymmetry, since in this case there are additional SUSY Killing spinors not contained in the ansatz (7).

Solutions with $\xi \neq 1$ describe globally regular $\mathcal{N} = 1$ deformations of the AdS. Their asymptotic form is determined by (15). Choosing the new radial coordinate $\tilde{r} = r/\sqrt{1+\delta}$ with $\delta = \frac{1}{2}\,\xi b^2$ and setting $Q = w_*/\sqrt{\xi(1+\delta)}$, the metric asymptotically approaches

$$ds^2 = -N^2dt^2 + \frac{d\tilde{r}^2}{N^2} + (1+\delta)\,\tilde{r}^2d\Omega^2, \tag{16}$$

where $N^2 = 1 - \frac{2M}{\tilde{r}} + \frac{\xi\tilde{r}^2}{2}$ and $M = b(w_*^2 - 1)/(1+\delta)^{3/2}$. This is the Schwarzschild-AdS metric with an *excess* of the solid angle – the area of the 2-sphere of constant $\tilde{r}$ being $4\pi(1+\delta)\tilde{r}^2$ in this geometry. The excess parameter δ and the 'mass' M vanish only for $\xi = 1$, and they tend to infinity as $\xi \to 0$.

$\underline{\xi < 0.}$ Solutions in this case are of the 'bag of gold' type, since they have compact spatial sections with the S^3 topology. The range of the Schwarzschild function $r(\rho)$ is finite: it starts from zero at $\rho = 0$ ('north pole'), increases up to a maximal value at some $\rho_e > 0$ ('equator'), and then decreases down to zero at some $\rho_* > \rho_e$ ('south pole'). Since $N \sim r' = 0$ at the equator, Eqs. (9)–(11) become singular at this point. To desingularize them, we set $P = wS$ thus obtaining $N = w\sqrt{1+S^2} = we^{V+\phi}$ (having chosen $Q = 1$ in (9)). Using this in (9)–(11)

reduces the system to

$$\begin{aligned} r' &= we^{V+\phi+\lambda}, \qquad \phi' = \sqrt{2}BSe^{\lambda-V-\phi}, \\ w' &= -rBe^{\lambda-V-2\phi}, \qquad V'-\phi' = \frac{\xi}{\sqrt{2}}Se^{\lambda-V}, \end{aligned} \tag{17}$$

with $S = \sqrt{e^{2V+2\phi}-1}$ and $B = -\frac{wS}{\sqrt{2}r} + \frac{1}{2}e^{-\phi}$. In addition, the relation $P = wS$ with P defined after Eqs. (11) gives the first integral for these equations. Eqs. (17) are completely regular at the equator, whose position coincides with zero of w. Imposing the gauge condition $\lambda = 0$ and demanding the solution to be regular at the 'north pole' gives $r = \rho + O(\rho^3)$, $w = 1 + O(\rho^2)$, $\phi = O(\rho^2)$, $V = O(\rho^2)$ for small ρ. At the 'south pole' we find the formal power series solution to be generically

$$\begin{aligned} r = 3w_*\beta_*\, x + O(x^3),\ w = w_* + \frac{w_*\kappa}{8}\, x^4 + O(x^6), \\ e^{-\phi} = \frac{|\kappa|x}{3\sqrt{2}} + O(x^3),\ e^{V-\phi} = \nu_* + \frac{\xi\beta_*\kappa}{4}\, x^2 + O(x^4), \end{aligned} \tag{18}$$

with $x = (\rho - \rho_*)^{1/3}$. Here ρ_*, $w_* < 0$, $\beta_* > 0$ and ν_* are integration constants, and $\kappa = (1-w_*^2)/(w_*^2\beta_*^2)$.

Solutions of Eqs. (17) in the interval $\rho \in [0, \rho_*]$ comprise a one-parameter family labeled by ξ. These solutions are regular for $\rho < \rho_*$, while at $\rho = \rho_*$ the dilaton diverges and the curvature is singular too. For any ξ, the profile of these solutions is qualitatively similar to the one shown in Fig. 2. As $\xi \to 0$, one has $\rho_* \to \infty$, $r(\rho_e) \to \infty$, and the solutions approach pointwise the CV monopole.

For one special value, $\xi = -2$, one has $w_* = -1$ and the expansions (18) are no longer valid. However, the solution can then be obtained analytically: $\phi = V = P = S = 0$, $w = \cos\rho$, $r = \sqrt{2}\sin\rho$, $e^\lambda = \sqrt{2}$. This solution is *globally* regular, also at the south pole, the spatial geometry being that of the round S^3. One can write down the metric and gauge field as

$$ds^2 = -dt^2 + 2\theta^a\theta^a, \qquad A^a = \theta^a, \tag{19}$$

where θ^a are invariant forms on S^3, $d\theta^a + \epsilon_{abc}\theta^b \wedge \theta^c = 0$. However, there is no SUSY enhancement in this case, and so $\mathcal{N} = 1$.

We have thus obtained the generalizations of the CV monopole (14) that comprise a two-parameter family labeled by ξ and ϕ_0. Although we have described explicitly only solutions with $\phi_0 = 0$, those with $\phi_0 \neq 0$ can be obtained by using the symmetry (13). The solutions

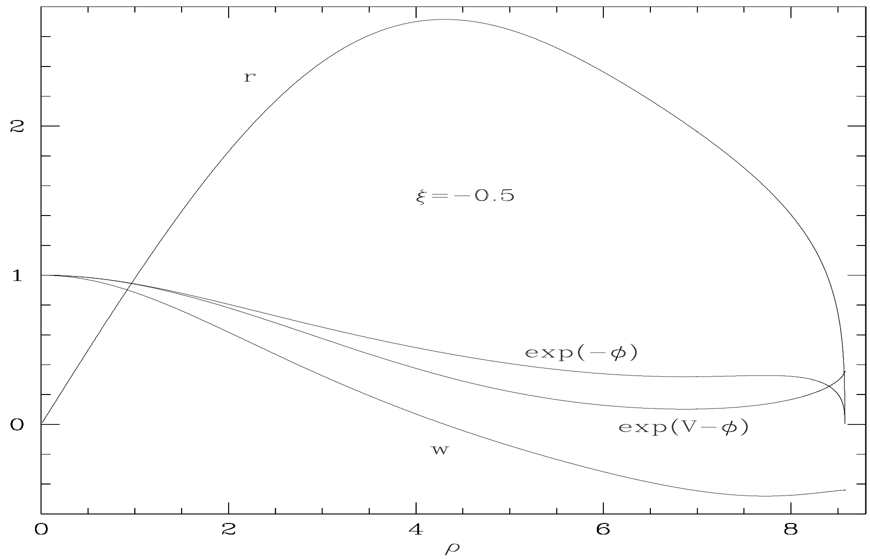

FIG. 2: The compact solutions with $\xi < 0$. They are generically singular at the 'south pole' where r vanishes and ϕ diverges.

generically have $\mathcal{N} = 1$, while for $\xi = 1$ the sypersymmetry is enhanced up to $\mathcal{N} = 4$. We know that the $\xi = 0$ solution can be uplifted to D=10 to become a vacuum of string theory [2]. It turns out that solutions with $\xi \neq 0$ can be uplifted to $D = 11$ to become vacua of M-theory.

The derivation of the GZ model via dimensional reduction of D=11 SUGRA was considered in Ref. [6]. Using formulas given in there combined with the symmetry (13), every D=4 vacuum (ds_4^2, A^a, ϕ) considered above maps to the M-theory solution $(ds_{(11)}^2, F_{[4]})$. The metric in D=11 is given by $ds_{(11)}^2 = |\xi|\Delta^{\frac{2}{3}} ds_4^2 + 8\Delta^{-\frac{1}{3}} ds_{(7)}^2$ with

$$ds_{(7)}^2 = \Delta\, d\eta^2 + \frac{\mathbf{c}^2}{X} \sum_a (\theta_{(1)}^a - \frac{1}{2}A^a)^2 + \mathbf{s}^2 X \sum_a (\theta_{(2)}^a)^2. \tag{20}$$

Here $\Delta = \mathbf{s}^2/X + \mathbf{c}^2 X$ with $1/X = \sqrt{|\xi|}e^{\phi}$, and $\theta_{(\iota)}^a$ $(\iota = 1, 2)$ are invariant forms on two different 3-spheres, $d\theta_\iota^a + \epsilon_{abc}\theta_\iota^b \wedge \theta_\iota^c = 0$. The case $\xi > 0$ corresponds to the reduction on S^7, one has then $\mathbf{c} = \cos\eta$, $\mathbf{s} = \sin\eta$ with $\eta \in [0, \pi/2]$, while for $\xi < 0$ one reduces on the $\mathcal{H}^{(2,2)}$ hyperbolic space [6], in which case $\mathbf{c} = \cosh\eta$, $\mathbf{s} = \sinh\eta$ and $\eta \in [0, \infty)$. The 4-form in D=11 reads

$$F_{[4]} = \sqrt{2}\mathbf{sc} * d\phi \wedge d\eta - \left(\frac{\xi}{|\xi|} \mathbf{c}^2 X^2 + \frac{\mathbf{s}^2}{X^2} + 2 \right) \frac{\epsilon_{(4)}}{\sqrt{2}}, \tag{21}$$

where $\epsilon_{(4)}$ is the 4-volume form and $*$ is the Hodge dual in the 4-space.

These formulas suggest a holographic interpretation for our solutions. For $\xi = 0$, according to [3], the dual theory is $D = 4$, $\mathcal{N} = 1$ SYM. For $\xi = 1$ we have $\phi = A^a = 0$, and the D=11 geometry is $AdS_4 \times S^7$. This is the near-horizon limit of the M2 brane, and the dual theory is therefore $D = 3$, $\mathcal{N} = 4$ SYM. This suggests that other solutions with positive $\xi \neq 1$ may describe some $\mathcal{N} = 1$ deformations of this theory. It would also be interesting to work out an interpretation for the compact solutions with $\xi < 0$, especially for the one given by Eq. (19).

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[1] O. Aharony, S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Phys.Rep.*, **232** (2000) 183.

[2] A.H. Chamseddine and M.S. Volkov, *Phys.Rev.Lett.*, **79** (1997) 3343; *Phys.Rev.*, **D57** (1998) 6242.

[3] J. Maldacena and C. Nunez, *Phys.Rev.Lett.*, **86** (2001) 588.

[4] D.Z. Freedman and J.H. Schwarz, *Nucl.Phys.*, **B137** (1978) 333.

[5] S.J. Gates and B. Zwiebach, *Phys.Lett.*, **B123** (1983) 200; B. Zwiebach, *Nucl.Phys.*, **B238** (1984) 367.

[6] M. Cvetic, H. Lu and C.N. Pope, *Nucl.Phys.*, **B574** (2000) 761; M. Cvetic, G.W. Gibbons and C.N. Pope, `hep-th/0401151`.